

Mathematics

K-8 Content Standards

Kindergarten

It is essential that these standards be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

K.1 Number and Operations and Algebra: **Represent, compare, and order whole numbers, and join and separate sets.**

- K.1.1 Read and write whole numbers to 10.
- K.1.2 Connect numbers, including written numerals, to the quantities they represent, using various physical models and representations.
- K.1.3 Count forward by ones beginning with any number less than 30; count backward by ones beginning with any number 10 or less.
- K.1.4 Recognize the number of objects in a small set (such as the arrangements of dots on a number cube) without counting.
- K.1.5 Count objects in a set using one-to-one correspondence and produce sets of given sizes.
- K.1.6 Compare and order sets or numerals by using both cardinal and ordinal meanings.
- K.1.7 Model simple joining and separating situations and represent them with objects, pictures, and/or numerals.
- K.1.8 Choose, combine, and apply effective strategies for solving joining and separating problems.
- K.1.9 Identify, duplicate, and extend simple number patterns and sequential and growing patterns (e.g., patterns made with shapes).

K.2 Geometry: **Describe shapes and space.**

- K.2.1 Identify, name, and describe basic two-dimensional shapes (e.g., square, circle, triangle, rectangle, regular hexagon) presented in a variety of ways (e.g., with different sizes or orientations).
- K.2.2 Identify, name, and describe basic three-dimensional shapes (e.g., sphere, cube, and cylinder).
- K.2.3 Use basic shapes and spatial reasoning to describe and model objects in their environment, and to construct more complex shapes.

K.3 Measurement: **Compare and order objects by attributes.**

- K.3.1 Identify the measurable attributes (e.g., length, weight) and non-measurable attributes (e.g., color) of an object.
- K.3.2 Compare, sort, and order objects according to measurable (e.g., longest to shortest, lightest to heaviest) and non-measurable (e.g., color, texture) attributes.
- K.3.3 Compare the lengths of two objects both directly (by comparing them with each other) and indirectly (by comparing both with a third object).

Grade 1

It is essential that these standards be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

1.1 Number and Operations: **Develop an understanding of whole number relationships, including grouping in tens and ones.**

- 1.1.1 Compare and order whole numbers to 100.
- 1.1.2 Represent whole numbers on a number line, demonstrating an understanding of the sequential order of the counting numbers and their relative magnitudes.
- 1.1.3 Count and group objects in tens and ones.
- 1.1.4 Identify the number of tens and ones in whole numbers between 10 and 100, especially recognizing the numbers 10 to 19 as 1 group of ten and a particular number of ones.
- 1.1.5 Determine the value of collections of pennies, nickels, and dimes.

1.2 Number and Operations and Algebra: **Develop understandings of addition and subtraction and strategies for basic addition facts and related subtraction facts.**

- 1.2.1 Model “part-whole,” “adding to,” “taking away from,” and “comparing” situations to develop an understanding of the meanings of addition and subtraction.
- 1.2.2 Develop and use efficient strategies for adding and subtracting whole numbers using a variety of models, including discrete objects, length-based models (e.g., lengths of connecting cubes) and number lines.
- 1.2.3 Apply with fluency sums to 10 and related subtraction facts.
- 1.2.4 Use the concept of commutative [$4 + 2 = 2 + 4$], associative [$(4 + 3) + 7 = 4 + (3 + 7)$], and identity [$0 + 3 = 3$] properties of addition to solve problems involving basic facts.
- 1.2.5 Relate addition and subtraction as inverse operations.
- 1.2.6 Identify, create, extend, and supply a missing element in number patterns involving addition or subtraction by a single-digit number.

1.3 Geometry: **Compose and decompose two- and three-dimensional geometric shapes.**

- 1.3.1 Describe geometric attributes of shapes (e.g., round, corners, sides) to determine how they are alike and different.
- 1.3.2 Recognize and create shapes that are congruent or have symmetry.
- 1.3.3 Compose and decompose shapes (e.g., cut a square into two right triangles and put two cubes together to make a rectangular prism), thus building an understanding of part-whole relationships as well as the properties of the original and composite shapes.
- 1.3.4 Recognize shapes when viewed from different perspectives and orientations.

Grade 2

It is essential that these standards be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

2.1 Number and Operations: **Develop an understanding of the base-ten numeration system and place-value concepts.**

- 2.1.1 Write, compare, and order whole numbers to 1000.
- 2.1.2 Understand and apply base-ten numeration, and count in multiples of one, two, five, ten, and one hundred.
- 2.1.3 Compose and decompose whole numbers less than one thousand by place value (e.g., 426 as 4 hundreds + 2 tens + 6 ones and $400 + 20 + 6$).
- 2.1.4 Use place value and properties of operations to find and use equivalent representations of numbers (such as 35 represented by 35 ones, 3 tens and 5 ones, or 2 tens and 15 ones).

2.2 Number and Operations and Algebra: **Develop fluency with addition facts and related subtraction facts, and with multi-digit addition and subtraction.**

- 2.2.1 Apply, with fluency, sums to 20 and related subtraction facts.
- 2.2.2 Solve multi-digit whole number problems by applying various meanings (e.g., taking away, and comparing) and models (e.g., combining or separating sets, using number lines, and hundreds charts) of addition and subtraction.
- 2.2.3 Develop fluency with efficient procedures for adding and subtracting multi-digit whole numbers and understand why the procedures work on the basis of place value and number properties.
- 2.2.4 Select and apply efficient methods to estimate sums and differences or calculate them mentally depending on the numbers and context involved.
- 2.2.5 Determine the value of mixed collections of coins to \$1.00.

2.3 Measurement: **Develop an understanding of linear measurement and facility in measuring.**

- 2.3.1 Determine length by finding the total number of equal-length units that are placed end-to-end without gaps or overlaps.
- 2.3.2 Apply concepts of partitioning (the mental activity of slicing the length of an object into equal-sized units) and transitivity (e.g., if object A is longer than object B and object B is longer than object C, then object A is longer than object C).
- 2.3.3 Demonstrate an understanding that using different measurement units will result in different numerical measurements for the same object.
- 2.3.4 Explain the need for equal length units and the use of standard units of measure.
- 2.3.5 Use rulers and other measurement tools to estimate and measure length in common units (e.g., centimeter and inch).
- 2.3.6 Use the measurement process: choose an appropriate measurement unit, compare that unit to the object, and report the number of units.
- 2.3.7 Demonstrate an understanding of time and use of time relationships (e.g., how many minutes in an hour, days in a week, and months in a year).
- 2.3.8 Tell time in increments of five minutes using analog and digital clocks.

Grade 3

It is essential that these standards be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

3.1 Number and Operations: **Develop an understanding of fractions and fraction equivalence.**

- 3.1.1 Represent common fractions (e.g., halves, thirds, fourths, tenths) as equal parts of a whole, parts of a set, or points or distances on a number line.
- 3.1.2 Recognize and demonstrate that sizes of fractional parts are relative to the size of the whole.
- 3.1.3 Use fractions to represent numbers that are equal to, less than, or greater than one.
- 3.1.4 Solve problems that involve comparing and ordering fractions by using models, benchmarks (0, $\frac{1}{2}$, 1), or common numerators or denominators.
- 3.1.5 Identify equivalent fractions using models, including the number line.
- 3.1.6 Add common fractions with like denominators.

3.2 Number and Operations, Algebra, and Data Analysis: **Develop understandings of multiplication and division, and strategies for basic multiplication facts and related division facts.**

- 3.2.1 Represent and apply the concept of multiplication as repeated addition.
- 3.2.2 Represent and apply the concept of division as repeated subtraction and forming equal groups.
- 3.2.3 Apply models of multiplication (e.g., equal-sized groups, arrays, area models, equal “jumps” on number lines and hundreds charts) and division (e.g., repeated subtraction, partitioning, and sharing) to solve problems.
- 3.2.4 Apply increasingly sophisticated strategies based on the number properties (e.g., place value, commutative, associative, distributive, identity, and zero) to solve multiplication and division problems involving basic facts.
- 3.2.5 Apply the inverse relationship between multiplication and division (e.g., $5 \times 6 = 30$, $30 \div 6 = 5$) and the relationship between multiples and factors.
- 3.2.6 Represent, analyze and extend number patterns using rules that involve multiplication and/or addition (e.g., $\{3, 6, 9, 12, \dots\}$, $\{1, 2, 4, 8, \dots\}$).
- 3.2.7 Analyze frequency tables, bar graphs, picture graphs, and line plots; and use them to solve problems involving addition, subtraction, multiplication, and division.

3.3 Geometry and Measurement: **Describe and analyze properties of two-dimensional shapes, including perimeters.**

- 3.3.1 Identify right angles in two-dimensional shapes and determine if angles are greater than or less than a right angle (obtuse and acute).
- 3.3.2 Identify, describe, compare, analyze, and informally classify triangles by their sides and angles.
- 3.3.3 Identify, describe, compare, analyze, and classify quadrilaterals (square, rectangle, parallelogram, rhombus, and trapezoid) by their sides and angles.
- 3.3.4 Identify, describe, and compare pentagons, hexagons, and octagons by the number of sides or angles.
- 3.3.5 Investigate and describe the results of decomposing, combining, and transforming polygons to make other polygons.
- 3.3.6 Build, draw, and analyze two-dimensional shapes to understand attributes and properties of two-dimensional space.

- 3.3.7 Determine an appropriate unit, tool, or strategy to find the perimeter of polygons.
- 3.3.8 Use attributes and properties of two-dimensional shapes to solve problems including applications involving parallel and perpendicular lines, congruence, symmetry, and perimeter.

Grade 4

It is essential that these standards be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

4.1 Number and Operations: **Develop an understanding of decimals, including the connections between fractions and decimals.**

- 4.1.1 Extend the base-ten system to read, write, and represent decimal numbers (to the hundredths) between 0 and 1, between 1 and 2, etc.
- 4.1.2 Use models to connect and compare equivalent fractions and decimals.
- 4.1.3 Determine decimal equivalents or approximations of common fractions.
- 4.1.4 Compare and order fractions and decimals.
- 4.1.5 Estimate decimal or fractional amounts in problem solving.
- 4.1.6 Represent money amounts to \$10.00 in dollars and cents, and apply to situations involving purchasing ability and making change.

4.2 Number and Operations and Algebra: **Develop fluency with multiplication facts and related division facts, and with multi-digit whole number multiplication.**

- 4.2.1 Apply with fluency multiplication facts to 10 times 10 and related division facts.
- 4.2.2 Apply understanding of models for multiplication (e.g., equal-sized groups, arrays, area models, equal intervals on the number line), place value, and properties of operations (commutative, associative, and distributive).
- 4.2.3 Select and use appropriate estimation strategies for multiplication (e.g., use benchmarks, overestimate, underestimate, round) to calculate mentally based on the problem situation when computing with whole numbers.
- 4.2.4 Develop and use accurate, efficient, and generalizable methods to multiply multi-digit whole numbers.
- 4.2.5 Develop fluency with efficient procedures for multiplying multi-digit whole numbers and justify why the procedures work on the basis of place value and number properties.

4.3 Measurement: **Develop an understanding of area and determine the areas of two-dimensional shapes.**

- 4.3.1 Recognize area as an attribute of two-dimensional regions.
- 4.3.2 Determine area by finding the total number of same-sized units of area that cover a shape without gaps or overlaps.
- 4.3.3 Recognize a square that is one unit on a side as the standard unit for measuring area.
- 4.3.4 Determine the appropriate units, strategies, and tools to solving problems that involve estimating or measuring area.
- 4.3.5 Connect area measure to the area model used to represent multiplication and use this to justify the formula for area of a rectangle.
- 4.3.6 Find the areas of complex shapes that can be subdivided into rectangles.
- 4.3.7 Solve problems involving perimeters and areas of rectangles and squares.
- 4.3.8 Recognize that rectangles with the same area can have different perimeters and that rectangles with the same perimeter can have different areas.

Grade 5

It is essential that these standards be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

5.1 Number and Operations and Data Analysis: **Develop an understanding of and fluency with addition and subtraction of fractions and decimals.**

- 5.1.1 Use fraction models to represent the addition and subtraction of fractions with unlike denominators.
- 5.1.2 Use decimal models, place value, and number properties to add and subtract decimals (to the thousandths).
- 5.1.3 Select and use appropriate strategies to estimate fraction and decimal sums and differences.
- 5.1.4 Develop fluency with efficient procedures for adding and subtracting fractions and decimals and justify why the procedures work.
- 5.1.5 Solve problems involving the addition and subtraction of fractions and decimals.
- 5.1.6 Use ordered pairs on coordinate graphs to specify locations and describe paths.
- 5.1.7 Construct and analyze double bar, line, and circle graphs to solve problems involving fractions and decimals.

5.2 Number and Operations and Algebra: **Develop an understanding of and fluency with division of whole numbers.**

- 5.2.1 Apply understanding of models for division (e.g., equal-sized groups, arrays, area models, equal intervals on the number line) and the relationship of division to multiplication to solve problems.
- 5.2.2 Apply concepts of place value and the properties of operations to solve problems involving division.
- 5.2.3 Select and use appropriate estimation strategies for division (e.g., use benchmarks, overestimate, underestimate, round) to calculate mentally based on the problem situation when computing with whole numbers.
- 5.2.4 Develop and use accurate, efficient, and generalizable methods to find quotients for multi-digit division problems.
- 5.2.5 Develop fluency with efficient procedures for dividing whole numbers and justify why the procedures work on the basis of place value and number properties.
- 5.2.6 Determine the most appropriate form of the quotient and interpret the remainder in a problem situation.

5.3 Geometry, Measurement, and Algebra: **Describe and relate two-dimensional shapes to three-dimensional shapes and analyze their properties, including volume and surface area.**

- 5.3.1 Identify and classify triangles by their angles (acute, right, obtuse) and sides (scalene, isosceles, equilateral).
- 5.3.2 Find and justify relationships among the formulas for the areas of triangles and parallelograms.
- 5.3.3 Describe three-dimensional shapes (triangular and- rectangular prisms, cube, triangular- and square-based pyramids, cylinder, cone, and sphere) by the number of edges, faces, and/or vertices as well as types of faces.
- 5.3.4 Recognize volume as an attribute of three-dimensional space.
- 5.3.5 Determine volume by finding the total number of same-sized units of volume that fill a three-dimensional shape without gaps or overlaps.

- 5.3.6 Recognize a cube that is one unit on an edge as the standard unit for measuring volume.
- 5.3.7 Determine the appropriate units, strategies, and tools for solving problems that involve estimating or measuring volume.
- 5.3.8 Decompose three-dimensional shapes and find surface areas and volumes of triangular and rectangular prisms.
- 5.3.9 Identify and measure necessary attributes of shapes to use area , surface area, and volume formulas to solve problems (e.g., to find which of two gift boxes needs the most wrapping paper or has the greater volume?).

Grade 6

It is essential that these standards be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

6.1 Number and Operations: **Develop an understanding of and fluency with multiplication and division of fractions and decimals.**

- 6.1.1 Select and use appropriate strategies to estimate fraction and decimal products and quotients.
- 6.1.2 Use and analyze a variety of strategies, including models, for solving problems with multiplication and division of fractions.
- 6.1.3 Use and analyze a variety of strategies, including models, for solving problems with multiplication and division of decimals.
- 6.1.4 Develop fluency with efficient procedures for multiplying and dividing fractions and decimals and justify why the procedures work.
- 6.1.5 Apply the inverse relationship between multiplication and division to make sense of procedures for multiplying and dividing fractions and justify why they work.
- 6.1.6 Apply the properties of operations to simplify calculations.
- 6.1.7 Use the relationship between common decimals and fractions to solve problems including problems involving measurement.

6.2 Number and Operations and Probability: **Connect ratio, rate, and percent to multiplication and division.**

- 6.2.1 Develop, analyze, and apply the meaning of ratio, rate, and percent to solve problems.
- 6.2.2 Determine decimal and percent equivalents for common fractions, including approximations.
- 6.2.3 Understand the meaning of probability and represent probabilities as ratios, decimals, and percents.
- 6.2.4 Determine simple probabilities, both experimental and theoretical.
- 6.2.5 Develop the concept of π as the ratio of the circumference of a circle to its diameter.

6.3 Algebra: **Write, interpret, and use mathematical expressions and equations.**

- 6.3.1 Use order of operations to simplify expressions that may include exponents and grouping symbols.
- 6.3.2 Develop the meanings and uses of variables.
- 6.3.3 Write, evaluate, and use expressions and formulas to solve problems.
- 6.3.4 Identify and represent equivalent expressions (e.g., different ways to see a pattern).
- 6.3.5 Represent, analyze, and determine relationships and patterns using tables, graphs, words and when possible, symbols.
- 6.3.6 Recognize that the solutions of an equation are the values of the variables that make the equation true.
- 6.3.7 Solve one-step equations by using number sense, properties of operations, and the idea of maintaining equality on both sides of an equation.

Grade 7

It is essential that these standards be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

7.1 Number and Operations and Algebra: **Develop an understanding of operations on all rational numbers and solving linear equations.**

- 7.1.1 Develop, analyze, and apply models (including everyday contexts), strategies, and procedures to compute with integers, with an emphasis on negative integers.
- 7.1.2 Extend knowledge of integers and positive rational numbers to solve problems involving negative rational numbers.
- 7.1.3 Develop and use strategies to estimate the result of rational number computations and justify the reasonableness of results.
- 7.1.4 Apply properties of rational numbers and algebra to write and solve linear equations in one variable.

7.2 Number and Operations, Algebra and Geometry: **Develop an understanding of and apply proportionality, including similarity.**

- 7.2.1 Represent proportional relationships with coordinate graphs and tables, and identify unit rate as the slope of the related line.
- 7.2.2 Apply ratio and proportionality to solve problems, including percent and simple probability.
- 7.2.3 Use coordinate graphs, tables, and equations to distinguish proportional relationships from other relationships, including inverse proportionality.
- 7.2.4 Develop and use scale factors and proportional relationships to solve problems, including similarity and congruence.
- 7.2.5 Convert among different units of measurement to solve problems, including rates.
- 7.2.6 Apply scale factor to analyze how the change in one measure (e.g., length, area, volume) affects another.

7.3 Measurement and Geometry: **Develop an understanding of and use formulas to determine surface area and volume.**

- 7.3.1 Use models to explain the reasonableness of formulas for the circumference and area of circles.
- 7.3.2 Know common estimates of π and use these values to estimate and calculate the circumference and area of a circle.
- 7.3.3 Solve problems involving areas and circumferences of circles.
- 7.3.4 Use models to explain the reasonableness of formulas for the surface area of pyramids and cylinders, and volume of pyramids, cylinders, and cones.
- 7.3.5 Find and justify relationships among the formulas for the areas of different polygons when determining surface area.
- 7.3.6 Solve problems involving surface areas of pyramids and cylinders and volumes of pyramids, cylinders, and cones.
- 7.3.7 Estimate and compute the area and volume of complex or irregular shapes by dividing them into basic shapes.

Grade 8

It is essential that these standards be addressed in contexts that promote problem solving, reasoning, communication, making connections, and designing and analyzing representations.

8.1 Algebra: Analyze and represent linear functions, and solve linear equations and systems of linear equations.

- 8.1.1 Translate among contextual, verbal, tabular, graphical, and algebraic representations of linear functions.
- 8.1.2 Determine the slope of a line and understand that it is a constant rate of change.
- 8.1.3 Identify and interpret the properties (i.e. slope, intercepts, continuity, and discreteness) of linear relationships as they are shown in the different representations and recognize proportional relationships ($y/x = k$ or $y = kx$) as a special case.
- 8.1.4 Use linear functions and equations to represent, analyze and solve problems, and to make predictions and inferences.
- 8.1.5 Relate systems of two linear equations in two variables and their solutions to pairs of lines that are intersecting, parallel, or the same line.
- 8.1.6 Use informal strategies (e.g., graphs or tables) to solve problems involving systems of linear equations in two variables.

8.2 Data Analysis and Algebra: Analyze and summarize data sets.

- 8.2.1 Organize and display data (e.g., histograms, box-and-whisker plots, scatter plots) to pose and answer questions; and justify the reasonableness of the choice of display.
- 8.2.2 Use measures of center and spread to summarize and compare data sets.
- 8.2.3 Interpret and analyze displays of data and descriptive statistics.
- 8.2.4 Compare descriptive statistics and evaluate how changes in data affect those statistics.
- 8.2.5 Describe the strengths and limitations of a particular statistical measure, and justify or critique its use in a given situation.
- 8.2.6 Use sample data to make predictions regarding a population.
- 8.2.7 Identify claims based on statistical data and evaluate the reasonableness of those claims.
- 8.2.8 Use data to estimate the likelihood of future events and evaluate the reasonableness of predictions.

8.3 Geometry and Measurement: Analyze two- and three-dimensional spaces and figures by using distance and angle.

- 8.3.1 Use properties of parallel lines, transversals, and angles to find missing sides and angles, and to solve problems including determining similarity or congruence of triangles.
- 8.3.2 Use models to show that the sum of the angles of any triangle is 180 degrees and apply this fact to find unknown angles.
- 8.3.3 Use models and logical arguments to show that the sum of the angles of any quadrilateral is 360 degrees, and apply this fact to find unknown angles.
- 8.3.4 Use models to explore the validity of the Pythagorean Theorem, and use it to find missing lengths.
- 8.3.5 Apply the Pythagorean Theorem to find distances in a variety of 2- and 3-dimensional contexts, including distances on coordinate graphs.
- 8.3.6 Use models and referents to explore and estimate square roots.