BLOCK 1 ~ LINEAR EQUATIONS

EXPRESSIONS AND EQUATIONS

LESSON 1  ORDER OF OPERATIONS  -----------------------------------------------  3
LESSON 2  EVALUATING EXPRESSIONS  ---------------------------------------------  7
LESSON 3  THE DISTRIBUTIVE PROPERTY  -------------------------------------------  11
  Explore! Match Them Up
LESSON 4  SOLVING ONE-STEP EQUATIONS  ----------------------------------------  16
  Explore! Addition and Subtraction Equations
LESSON 5  SOLVING TWO-STEP EQUATIONS  ----------------------------------------  21
LESSON 6  SOLVING MULTI-STEP EQUATIONS  ---------------------------------------  26
  Explore! Multi-Step Equations
LESSON 7  THE COORDINATE PLANE AND SCATTER PLOTS  ---------------------------  32
  Explore! A Coordinate Plane Connect-the-Dots
REVIEW  BLOCK 1 ~ EXPRESSIONS AND EQUATIONS  --------------------------------  38

WORD WALL

LIKE TERMS  TERM  INVERSE OPERATIONS  ZERO PAIR
COORDINATE PLANE  ORIGIN  ORDER OF OPERATIONS
DISTRIBUTIVE PROPERTY  CONSTANT  EQUIVALENT EXPRESSIONS
AXES (x-axis and y-axis)  COEFFICIENT  EQUATION
ORDERED PAIR  QUADRANTS  SCATTER PLOT
ALGEBRAIC EXPRESSION  ABSOLUTE VALUE
**What’s the Process?**
Create a poster that explains the process of solving a multi-step equation.

*See page 25 for details.*

**Small Business Profits**
Study three different business start-up plans. Determine how many items the business will need to sell to break even and make a profit.

*See page 31 for details.*

**Order of Ops Poetry**
Write three different poems about the order of operations.

*See page 15 for details.*

**Like Terms Game**
Create a matching game requiring players to combine like terms to make pairs.

*See page 15 for details.*

**Scatter Plot Survey**
Conduct a survey and record your results on a scatter plot. Determine if there is a correlation between the items.

*See page 36 for details.*

**Temperature Systems**
Convert temperatures from one system to another.

*See page 25 for details.*

**Equation Mats**
Produce a “How To...” guide to show others how to use equation mats to solve a variety of equations.

*See page 31 for details.*

**Inequalities**
Solve inequalities and graph solutions on a number line.

*See page 37 for details.*

**Scientific Notation**
Learn to write very large and very small numbers in scientific notation.

*See page 6 for details.*
Mr. Marshall asked his students to find the value of the expression $7 + 2 \cdot 3 - 1$

Sasha is positive that the answer is 26. Michelle believes the answer is 12. Mr. Marshall has both students show their work on the board. Look at their work below. Who do you agree with? Why?

<table>
<thead>
<tr>
<th>Sasha</th>
<th>Michelle</th>
</tr>
</thead>
<tbody>
<tr>
<td>$7 + 2 \cdot 3 - 1$</td>
<td>$7 + 2 \cdot 3 - 1$</td>
</tr>
<tr>
<td>$= 9 \cdot 3 - 1$</td>
<td>$= 7 + 6 - 1$</td>
</tr>
<tr>
<td>$= 27 - 1$</td>
<td>$= 13 - 1$</td>
</tr>
<tr>
<td>$= 26$</td>
<td>$= 12$</td>
</tr>
</tbody>
</table>

Mathematicians have established an order of operations. The order of operations is a set of rules which are followed when evaluating an expression with more than one operation. Using the correct order of operations helped Michelle find the correct answer to Mr. Marshall’s question because she multiplied before adding or subtracting.

**Order of Operations**

1. Find the value of expressions inside grouping symbols such as parentheses, absolute value bars and fraction bars.
2. Find the value of all powers.
3. Multiply and divide from left to right.
4. Add and subtract from left to right.

All operations within grouping symbols must be completed before any of the other steps are completed. The three types of grouping symbols you will work with in this book are parentheses, absolute value bars and fraction bars.

**Absolute value** is the distance a number is from zero. The absolute value of a number is always positive. For example:

\[ |−9 + 4| = |−5| = 5 \]

The fraction bar is another symbol used to represent division. Perform all operations in the numerator and denominator before dividing. For example:

\[ \frac{2 + 10}{5 - 1} = \frac{12}{4} = 3 \]
EXAMPLE 1

Find the value of each expression.

a. \(8(−3 + 7) − 2 \cdot 5\)

b. \(5^2 + |3 − 8| ÷ 5\)

Solutions

a. Add the integers inside the parentheses. 
\[8(−3 + 7) − 2 \cdot 5 = 8(4) − 2 \cdot 5\]
Multiply from left to right.
\[= 32 − 10\]
Subtract.
\[= 22\]

b. Subtract inside the absolute value bars. 
\[5^2 + |3 − 8| ÷ 5 = 5^2 + |−5| ÷ 5\]
Make the value inside the bars positive.
\[= 5^2 + 5 ÷ 5\]
Find the value of the power.
\[= 25 + 1\]
Divide.
\[= 26\]

EXAMPLE 2

Find the value of the expression.

\[\frac{(-5 - 3)^2}{6 - 4} - 50\]

Solution

Perform operation inside the parentheses. 
\[\frac{(-5 - 3)^2}{6 - 4} - 50 = \frac{(-8)^2}{6 - 4} - 50\]
Find the value of \((-8)^2\).
\[= \frac{64}{2} - 50\]
Subtract in the denominator.
\[= 32 - 50\]
Divide numerator by denominator.
\[= -18\]

EXAMPLE 3

Jakim’s family took a vacation to California. The plane tickets cost a total of $840, the hotel cost $250 and gas cost $130. There are 5 people in Jakim’s family.

a. Write an expression that could be used to find the cost per person.

b. Find the cost per person.

Solutions

a. The numbers must be added before dividing by the number of people.
Use parentheses or the fraction bar to group the numbers that must be added.

\[(840 + 250 + 130) ÷ 5 \quad \text{OR} \quad \frac{840 + 250 + 130}{5}\]

b. \[\frac{840 + 250 + 130}{5} = \frac{1220}{5} = \$244\]

The vacation cost $244 per person.
1. Nathan and Takashi each evaluated the expression $16 - 5 \cdot 2 + 4$. Nathan believes the solution is 10. Takashi disagrees and says the answer is 26.
   a. Who is correct? What operation did he perform first?
   b. What operation was done first by the student who was incorrect?

Evaluate each expression.

2. $18 \div 2 + 7 \cdot 3$
3. $6 \cdot 5 - 4 \div 2$
4. $(3 + 5)^2 - 8 \cdot 3$
5. $45 \div (-3)^2 + 4(2 + 1)$
6. $5 + 4|2 + 8|$
7. $5(11 + 1) - 3(2 + 11)$
8. $12 - 2 \cdot 10 \div 5 - 1$
9. $|3 - 27| \div |4 + 2|$
10. $21 \div 3 \cdot 7 - 4^2$

Evaluate each expression.

11. $\frac{4 + 28}{6 - 2}$
12. $\frac{20(7 - 3)}{2 \cdot 5} - 7$
13. $\frac{|-18 + 3|}{9 - 6}$
14. $31 - \frac{5 + 4 \cdot 5 - 1}{2}$
15. $\frac{6(9 - 7)^2}{-2}$
16. $\frac{25}{5} - \frac{10|5 - 1|}{2}$

17. The Chess Club is selling tickets to Saturday’s Winter Ball. Admission with Student ID is $5 per person. Admission without a Student ID is $8. The Chess Club sold 130 tickets to students with an ID Card and 40 tickets to students without ID Cards.
   a. Write an expression to represent the total amount of money the Chess Club collected in ticket sales.
   b. How much money did the Chess Club collect?

   a. Write an expression that could be used to find the cost per person.
   b. Find the cost per person.

19. Explain why it is necessary to have an order of operations in mathematics.

20. Create an expression with at least five numbers and two different operations that has a value of 15.

Insert one set of parentheses in each numerical expression so that it equals the stated amount.

21. $6 + 3 + 11 \div 4 = 5$
22. $7 + 1 \cdot 4 - 2 \cdot 5 = 17$
23. $-1 \cdot 6 + 8 - 4 \div 2 + 2 = 1$
Tic-Tac-Toe ~ Scientific Notation

Scientific notation is a method used by scientists and mathematicians to express very large and very small numbers. Scientific notation is an exponential expression using a power of 10.

\[ N \times 10^P \]

Use the following process to convert a large or small number into scientific notation:

**Step 1**: Locate the decimal point and move it left or right so there is only one non-zero digit to its left. This number represents the value of \( N \).

**Step 2**: Count the number of places that you moved the decimal point in **Step 1**. This number represents the value of \( P \). If you move the decimal point to the left, the sign of \( P \) is positive. If you move the decimal point to the right, the sign of \( P \) is negative.

*For example:*

**A. Convert 52,000 to scientific notation.**

**Step 1**: Move the decimal point to the **left** so there is only one non-zero digit to its left.

\[ 52,000 \rightarrow 5.2 \]

**Step 2**: Count how many places the decimal point was moved in the number above.

\[ 5 \, 2 \, 0 \, 0 \, 0 \rightarrow 4 \, \text{places} \]

The decimal point was moved 4 places to the **left**. Since the decimal point was moved left, the \( P \) value is positive. Scientific notation for 52,000 is \( 5.2 \times 10^4 \).

**B. Convert 0.00492 to scientific notation.**

**Step 1**: Move the decimal point to the **right** so there is only one non-zero digit to its left.

\[ 0.00492 \]

**Step 2**: Count how many places the decimal point was moved in the number above.

\[ 0. \, 0 \, 0 \, 4 \, 9 \, 2 \rightarrow 3 \, \text{places} \]

The decimal point was moved 3 places to the **right**. Since the decimal point was moved right, the \( P \) value is negative. Scientific notation for 0.00492 is \( 4.92 \times 10^{-3} \).

Write each large or small number in scientific notation.

1. 0.0049
2. 70,000
3. 5,930,000,000
4. 0.00821
5. 0.0000001
6. 320,000
7. 680
8. 0.00105
9. 75,000
10. When will numbers in scientific notation have a positive power of 10? When will the power of 10 be negative?
11. Keely wrote the number 9,200,000 in scientific notation. She incorrectly wrote the number as \( 92 \times 10^5 \). Explain what Keely did wrong. Write the number correctly in scientific notation.
12. According to most-expensive.net, *Spider-Man 3* cost more to produce than any other movie before it. The budget was $258,000,000. Write this number in scientific notation.
A lesson on evaluating expressions.

A lesson on evaluating expressions.

**Lesson 2**

**Evaluating Expressions**

1. Rewrite the expression by replacing the variables with the given values.
2. Follow the order of operations to compute the value of the expression.

**Example 1**

Evaluate each algebraic expression.

a. \(2x - 8\) when \(x = 4\)

b. \(-\frac{5(m + y)}{m}\) when \(m = -2\) and \(y = 10\)

**Solutions**

a. Write the expression.
   Substitute 4 for \(x\).
   Multiply.
   Subtract.

b. Write the expression.
   Substitute \(-2\) for \(m\) and 10 for \(y\).
   Add inside parentheses.
   Multiply.
   Divide the numerator by the denominator.

<table>
<thead>
<tr>
<th>Input, (x)</th>
<th>(\frac{4 + 5x}{2})</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>(\frac{4 + 5(2)}{2})</td>
<td>7</td>
</tr>
<tr>
<td>5</td>
<td>(\frac{4 + 5(5)}{2})</td>
<td>14.5</td>
</tr>
<tr>
<td>(-6)</td>
<td>(\frac{4 + (-6)}{2})</td>
<td>(-13)</td>
</tr>
</tbody>
</table>

Different values can be substituted for the variable in an expression. A table is used to help organize mathematical computations. The values for the variable are often called the input values. The values of the expression are often called the output values.
**EXAMPLE 2**  
Fill in the table by evaluating the given expression for the values listed.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-3x + 7$</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-3(-2) + 7</td>
<td>13</td>
</tr>
<tr>
<td>0</td>
<td>-3(0) + 7</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>-3(4) + 7</td>
<td>-5</td>
</tr>
<tr>
<td>9</td>
<td>-3(9) + 7</td>
<td>-20</td>
</tr>
</tbody>
</table>

**Solution**  
Rewrite the expression with the input value in the place of $x$ and then follow the order of operations. In this case, multiply and then add to find the output values.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$-3x + 7$</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>-3(-2) + 7</td>
<td>13</td>
</tr>
<tr>
<td>0</td>
<td>-3(0) + 7</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>-3(4) + 7</td>
<td>-5</td>
</tr>
<tr>
<td>9</td>
<td>-3(9) + 7</td>
<td>-20</td>
</tr>
</tbody>
</table>

An **equation** is a mathematical sentence that contains an equals sign between two expressions. Equations that involve at least one variable are neither true nor false until the equation is evaluated with given values for the variables. Values can be considered the solutions to an equation if they make the equation true.

**EXAMPLE 3**  
State whether each equation is true or false for the values of the variables given.

a. $3x + 2y = 8$ when $x = 2$ and $y = 1$

b. $-5x + 9 = y$ when $x = 6$ and $y = -21$

c. $y = \frac{1}{2}x + 1$ when $x = 8$ and $y = 17$

**Solutions**

a. Substitute $x = 2$ and $y = 1$.  
$3(2) + 2(1) \overset{?}{=} 8$
Multiply.  
$6 + 2 \overset{?}{=} 8$
Add.  
$8 = 8$ **TRUE**

b. Substitute $x = 6$ and $y = -21$.  
$-5(6) + 9 \overset{?}{=} -21$
Multiply.  
$-30 + 9 \overset{?}{=} -21$
Add.  
$-21 = -21$ **TRUE**

c. Substitute $x = 8$ and $y = 17$.  
$17 \overset{?}{=} \frac{1}{2}(8) + 1$
Multiply.  
$17 \overset{?}{=} 4 + 1$
Add.  
$17 \neq 5$ **FALSE**
Evaluate each expression when $x = 3$.

1. $2x - 1$
2. $\frac{-9x + 5}{2}$
3. $\frac{1}{3}x - 7$
4. $(x + 3)^2$
5. $5(x - 1)^2 + 2$
6. $-9 - 5x$

Evaluate each expression for the given values of the variables.

7. $\frac{3x - 5}{2}$ when $x = 7$
8. $\frac{-2(7x - 3)}{5}$ when $x = 4$
9. $8y - 2x$ when $y = 5$ and $x = 5$
10. $\frac{3}{4}a + 3$ when $a = 12$
11. $(4 + 3x)^2$ when $x = -1$
12. $-3y + -4m$ when $y = 1$ and $m = \frac{1}{2}$
13. $y + \frac{x - 7}{5}$ when $y = -2$ and $x = 22$
14. $0.5m - 0.1n$ when $m = 10$ and $n = 10$

Copy each table. Complete each table by evaluating the given expression for the values listed.

15. | $x$ | $6x - 4$ | Output |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{1}{2}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

16. | $x$ | $\frac{5x - 3}{2}$ | Output |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

State whether each equation is true or false for the values of the variables given.

17. $5x + 2y = 10$ when $x = 2$ and $y = 0$

18. $-3x + y = 7$ when $x = 1$ and $y = 4$

19. $y = 8x + 9$ when $x = 5$ and $y = 40$

20. $4y = x + 5$ when $x = 7$ and $y = 3$

21. $y = 3(x - 6)$ when $x = 4$ and $y = -6$

22. $0.5x + 5y = 17.5$ when $x = 5$ and $y = 4$

23. $-3y + -4x = 20$ when $x = 0$ and $y = -5$

24. $\frac{1}{2}x + \frac{1}{2}y = 4$ when $x = 2$ and $y = 8$
25. Use the formulas given to find the perimeter of each figure above when $w = 2$, $x = 5$, $y = 3$ and $z = 10$.

   a. TRIANGLE $P = x + y + z$

   b. RECTANGLE $P = 2(y + z)$

   c. TRAPEZOID $P = w + 2x + z$

26. Use the formulas given to find the area of each figure above when $w = 2$, $x = 5$, $y = 3$ and $z = 10$.

   a. TRIANGLE $A = \frac{1}{2} wz$

   b. RECTANGLE $A = yz$

   c. TRAPEZOID $A = \frac{1}{2} y(w + z)$

27. Tom went shopping at the mall. He found one type of shirt he liked for $12. He also discovered a pair of shorts for $16. Both the shirt and the shorts came in many different colors.

   a. Let $x$ represent the number of shirts and $y$ represent the number of shorts Tom purchases. Write an algebraic expression that represents the total cost for $x$ shirts and $y$ shorts.

   b. Tom decides to buy three shirts and five pairs of shorts. What is the total cost for this purchase?

   c. Sam, a friend of Tom’s, decides to buy the same kind of shorts and shirts. His total cost for his purchase was $80. How many shirts and pairs of shorts do you think he purchased?

28. The table at right shows admission prices for Centerville’s movie theater.

   a. The Johnson family consists of 2 adults, 1 senior citizen and three children (ages 3, 7 and 13). What will be the total cost for admission for the Johnson family to see a movie at the theater?

   b. Jacob is having a birthday bash for his thirteenth birthday. His mom agreed to take Jacob and 9 of his friends to the theater for the party. All of Jacob’s friends are also twelve or thirteen. How much will it cost for all the kids plus Jacob’s mom to go to the movie?

   c. Derrick spent $26.50 on movie admissions for his family. Give one possible description of the ages of people in Derrick’s family.

<table>
<thead>
<tr>
<th>Centerville Movie Admission</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Adult (18-61 years old)</td>
<td>$8.00</td>
</tr>
<tr>
<td>Senior Citizen (62 years and above)</td>
<td>$6.50</td>
</tr>
<tr>
<td>Children (5-17 years old)</td>
<td>$4.00</td>
</tr>
<tr>
<td>Children (4 years and under)</td>
<td>$2.50</td>
</tr>
</tbody>
</table>

REVIEW

Evaluate each expression.

29. $5 + 2 \cdot 7 - 20$

30. $\frac{6 + 9}{3} + 4$

31. $1 - 10 + -11 + 2 \cdot 3^2$

32. $3(2 + 4)^2 - 100$

33. $-5 \cdot 7 + 6(-2 - 1)$

34. $\frac{70 - 10}{6 + 4} - 6$
Every algebraic expression has at least one term. A term is a number or the product of a number and a variable. Terms are separated by addition and subtraction signs. A constant is a term that has no variable. The number multiplied by a variable in a term is called the coefficient.

Some expressions contain parentheses. One tool that will help you work with these expressions is called the Distributive Property. The Distributive Property allows you to rewrite an expression without parentheses. This is done by distributing the front coefficient to each term inside the parentheses. This will be a crucial step in solving equations that contain parentheses.

**The Distributive Property**

For any numbers $a$, $b$ and $c$:

$a(b + c) = a \cdot b + a \cdot c$

$a(b - c) = a \cdot b - a \cdot c$

**Example 1**

Use the Distributive Property to simplify each expression.

<table>
<thead>
<tr>
<th></th>
<th>a. $2(x + 6)$</th>
<th>b. $\frac{1}{4}(y - 20)$</th>
<th>c. $-5(3x - 1)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Solutions</strong></td>
<td>Drawing arrows from the front coefficient to each term inside the parentheses will help guide you.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>a. $2(x + 6)$</td>
<td>$= 2(x) + 2(6)$</td>
<td>$= 2x + 12$</td>
<td></td>
</tr>
<tr>
<td>b. $\frac{1}{4}(y - 20)$</td>
<td>$= \frac{1}{4}(y) - \frac{1}{4}(20)$</td>
<td>$= \frac{1}{4}y - 5$</td>
<td></td>
</tr>
<tr>
<td>c. $-5(3x - 1)$</td>
<td>$= -5(3x) - (-5)(1)$</td>
<td>$= -15x + 5$</td>
<td></td>
</tr>
</tbody>
</table>
The Distributive Property is very useful when doing mental math calculations. Certain numbers are easier to multiply together than others. In Example 2, notice how you can rewrite a number as a sum or difference of two other numbers that are easier to work with and then do the math mentally.

**Example 2**

Find each product by using the Distributive Property and mental math.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>b.</td>
</tr>
<tr>
<td>4(103)</td>
<td>998 · 7</td>
</tr>
</tbody>
</table>

**Solutions**

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>a.</td>
<td>b.</td>
</tr>
<tr>
<td>4(100) + 4(3)</td>
<td>7(1000) − 7(2)</td>
</tr>
<tr>
<td>400 + 12 = 412</td>
<td>7000 − 14 = 6,986</td>
</tr>
</tbody>
</table>

An algebraic expression or equation may have like terms that can be combined. Like terms are terms that have the same variable raised to the same power. The numerical coefficients do not need to be the same. If there are parentheses involved in the expression, the Distributive Property must be used FIRST before combining like terms.

An expression is simplified if it has no parentheses and all like terms have been combined. When combining like terms you must remember that the operation in front of the term (addition or subtraction) must remain attached to the term. Rewrite the expression by grouping like terms together before adding or subtracting the coefficients to simplify.

**Example 3**

Simplify by combining like terms. \(3x − 2y + 4 − 2x + x + 4y\)

**Solution**

Mark terms that are alike.

Group like terms.

Combine. A subtraction sign is treated as a negative sign on the coefficient it precedes.

**Example 4**

Simplify by combining like terms. \(-5(2x − 1) + 3x − 2\)

**Solution**

Distribute. \(-5(2x − 1) + 3x − 2 = −10x + 5 + 3x − 2\)

Group like terms. \(= −10x + 3x + 5 − 2\)

Combine. \(= −7x + 3\)

In each example so far in this lesson, the original and simplified expressions are called equivalent expressions. Two or more expressions that represent the same simplified algebraic expression are called **equivalent expressions**.
Step 1: On your own paper, copy each expression below. Leave at least two lines between each expression.

A. \(2(x + 4) - 5\)  
F. \(20 - 3(x + 4) - 2x\)  
B. \(3(x - 3) - 2x\)  
G. \(3x + 3 + 7x - 8x\)  
C. \(1 + 8(x + 1)\)  
H. \(-2(x - 3) + x + 5\)  
D. \(9 - 4x - 1 - x\)  
I. \(2(x + 7) - 3(x + 1)\)  
E. \(2(4x - 5) + 1 - 7x\)  
J. \(6x + 5x - x - 2x + 9\)

Step 2: Simplify each expression.

Step 3: Every expression listed above is equivalent to one other expression in the list. Classify the ten expressions into five groups of equivalent expressions.

Step 4: Create another ‘non-simplified’ expression for each group that is equivalent to the other expressions in the group.

**EXERCISES**

Use the Distributive Property to simplify each expression.

1. \(5(x + 1)\)  
2. \(\frac{3}{5}(5x + 10)\)  
3. \(2(4m + 5)\)  
4. \(-6(x - 10)\)  
5. \(-3(h - 11)\)  
6. \(16\left(\frac{1}{2}x - 2\right)\)

Find the product by using the Distributive Property.

7. \(7(105)\)  
8. \(68(10.5)\)  
9. \(896 \cdot 4\)  
10. \(6(999)\)

11. Tasha finds 7 DVDs she wants to purchase at the video store. Each DVD is $14.95.
   a. Show how Tasha could use the distributive property to help mentally calculate the total cost of the DVDs.
   b. How much will she pay for the seven DVDs?
Simplify each expression.

12. $9 + 2x - 4 + 8x$
13. $7(x - 2) + 6(x + 1)$
14. $7 + 3(x - 4)$
15. $-9x + 8x + 7y - 6y$
16. $7x + 3y - x + 4y - 2x$
17. $12y - 3x + 10x - y$
18. $-10(3x + 2) - 12$
19. $9x - 3(x + 2) + 7$

Write and simplify an expression for the perimeter of each figure.

20.

21.

22.

23.

Write and simplify an expression for the area of each figure.

Area of Rectangle = length \cdot width
Area of Triangle = \frac{1}{2}base \cdot height

24.

25.

26.

27.

In each set of three expressions, two are equivalent. Simplify each expression to find the equivalent expressions.

28. A. $3x + 4 - 2 + 7x$
   B. $18x - 5 - 8x + 7$
   C. $2x + 8x - 4 + 2$

29. A. $3(x - 1) + 12$
   B. $4(x + 3) - 1$
   C. $4(x + 2) - x + 1$

30. Whitney states that $8x - 7$ is equivalent to $7 - 8x$. Do you agree with her? Explain your answer.
Lesson 3 ~ The Distributive Property

31. Evaluate each expression when \( x = 2 \).
   a. \( 5(x + 8) \)
   b. \( \frac{3x - 2}{6} \)
   c. \( (x + 5)^2 + 2x \)
   d. \( 16\left(\frac{1}{2}x + 1\right) \)

32. Francis earns $20 per day for yard work plus $4 more for every pound of yard debris she removes.
   a. Write an expression that represents the total amount Francis would be paid for a day of yard work when she disposed of \( x \) pounds of yard debris.
   b. Calculate how much Francis would get paid on a day when she removed 18 pounds of yard debris.

**Tic-Tac-Toe ~ Order of Ops Poetry**

Poetry is an art form that is composed of carefully chosen words to express a greater depth of meaning. Poetry can be written about many different subjects, including mathematics. The order of operations is a key element in mathematics so that all students, teachers and mathematicians reach the same answer when calculating the value of the same expression. Write two different poems about the order of operations. One should be an acrostic poem and the other should be a quatrain. Research and find another style of poetry. Write one more poem about the order of operations using this style.

**Tic-Tac-Toe ~ Like Terms Game**

Write 15 variable expressions containing like terms that have not been combined. Make sure you have a minimum of five expressions containing parentheses. Cut thicker paper (such as cardstock, construction paper, index cards or poster board) into 30 equal-sized pieces. Write each expression on one card. On another card, write the expression in simplest form. Use these cards to play a memory game with a friend, classmate or family member. Record each pair of cards each participant wins on a sheet of paper by listing the non-simplified and simplified expressions. Turn in the cards and the game sheet to your teacher.
In order to solve a mathematical equation, the variable must be isolated on one side of the equation and have a front coefficient of one. This process is sometimes referred to as “getting the variable by itself”. The most important thing to remember is that the equation must always remain balanced. Whatever occurs on one side of the equals sign MUST occur on the other side so the equation remains balanced. The properties of equality are listed below. Note that you can perform any of the four basic operations to an equation as long as that operation is done to both sides of the equation.

**The Properties of Equality**

For any numbers $a$, $b$ and $c$:
- If $a = b$, then $a + c = b + c$ (Addition Property of Equality)
- If $a = b$, then $a - c = b - c$ (Subtraction Property of Equality)
- If $a = b$, then $a \cdot c = b \cdot c$ (Multiplication Property of Equality)
- If $a = b$, then $\frac{a}{c} = \frac{b}{c}$ (Division Property of Equality)

**EXPLORE! ADDITION AND SUBTRACTION EQUATIONS**

Each blue chip represents the integer $+1$. Each red chip represents the integer $-1$. When a positive integer chip is combined with a negative integer chip, the result is zero. This pair of integer chips is called a **zero pair**.

**Step 1:** On your equation mat, place the variable cube on one side with 4 positive integer chips. On the other side of the mat, place 9 positive integer chips. Write the equation that is represented by the items on the mat.

**Step 2:** In order to isolate the variable, you must get rid of the 4 integer chips with the variable. If you take chips off one side of the equation, you must do the same on the other side. How many chips are left on the right side? What does this represent?

**Step 3:** Clear your mat and place chips on the mat to represent the equation $x - 2 = 6$. Draw this on a sheet of paper.

**Step 4:** How could you “get rid of” the chips that are with the variable? How many chips end up on the opposite side of the mat when you cancel out the 2 negative integer chips that are with the variable? Write your answer in the form $x = \_\_\_$. 

Lesson 4 ~ Solving One - Step Equations
Step 5: Clear your mat and place chips on the mat to represent the equation \(-3 + x = -1\). Draw this on your own paper.

Step 6: Isolate the variable by canceling out chips on the variable side of the equation. Remember that whatever you do to one side of the equation you must do to the other side. What does \(x\) equal in this case?

Step 7: Create your own equation on your mat. Record the algebraic equation on your paper.

Step 8: Solve your equation. What does your variable equal?

Step 9: Write a few sentences that describe how to solve a one-step addition or subtraction equation using equation mats, integer counters and variable cubes.

You will not always have integer chips or an equation mat available to use when you are solving equations. You can solve equations using the balancing method on paper. When isolating the variable, remember to perform an inverse operation on both sides of the equation. Inverse operations are operations that undo each other, such as addition and subtraction.

**Example 1**

**Solve for \(x\). Check your solution.**

a. \(x - 6 = 22\)  
   **Solutions**

   Draw a vertical line through the equals sign to help you stay organized. Whatever is done on one side of the line to cancel out a value must be done on the other side.

   \[
   \begin{align*}
   a. \quad x - 6 &= 22 \\
   x + 6 &= 22 \\
   \checkmark \quad 28 - 6 &= 22 \\
   22 &= 22 \\
   \\
   b. \quad 8x &= 88 \\
   \frac{8x}{8} &= \frac{88}{8} \\
   x &= 11 \\
   \checkmark \quad 8(11) &= 88 \\
   88 &= 88 \\
   \\
   c. \quad x + 10 &= -7 \\
   x - 10 &= -7 \\
   \checkmark \quad -17 + 10 &= -7 \\
   -7 &= -7 \\
   \\
   d. \quad \frac{x}{5} &= 21 \\
   5x &= 105 \\
   \checkmark \quad \frac{105}{5} &= 21 \\
   21 &= 21
   \end{align*}
   \]
Equations in this lesson have been written symbolically. There will be times when an equation will be written in words and need to be translated into mathematical symbols in order to solve the equation. Here are some key words you need to remember when translating words into math symbols:

**Findings**

<table>
<thead>
<tr>
<th>Addition</th>
<th>Subtraction</th>
<th>Multiplication</th>
<th>Division</th>
<th>Equals</th>
</tr>
</thead>
<tbody>
<tr>
<td>sum</td>
<td>difference</td>
<td>product</td>
<td>quotient</td>
<td>is equal to</td>
</tr>
<tr>
<td>increased by</td>
<td>decreased by</td>
<td>multiplied by</td>
<td>divided by</td>
<td></td>
</tr>
<tr>
<td>more than</td>
<td>less than</td>
<td>times</td>
<td></td>
<td></td>
</tr>
<tr>
<td>plus</td>
<td>minus</td>
<td>of</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Example 2**

Write an equation for each statement. Solve each equation and check your solution.

a. The product of eleven and a number is seventy-seven.

b. The sum of a number and 52 is 98.

c. A number decreased by 13 is 214.

**Solution**

a. Write the equation. “Product” means multiplication.

\[11x = 77\]

Divide both sides of the equation by 11.

\[x = \frac{77}{11} = 7\]

✓ Check the answer.

\[11 \times 7 = 77\]

b. Write the equation. “Sum” means addition.

\[p + 52 = 98\]

Subtract 52 from both sides of the equation.

\[p = 46\]

✓ Check the answer.

\[46 + 52 = 98\]

c. Write the equation. “Decreased by” means subtraction.

\[y - 13 = 214\]

Add 13 to both sides of the equation.

\[y = 227\]

✓ Check the answer.

\[227 - 13 = 214\]

**Example 3**

The sum of Kirk’s age and his dad’s age is 53. Kirk is 14.

Write an equation that represents this situation using \(d\) to represent the dad’s age. Solve the equation and check your solution.

**Solution**

Write the equation.

\[14 + d = 53\]

Subtract 14 from both sides of the equation.

\[d = 39\]

✓ Check the answer.

\[14 + 39 = 53\]

Kirk’s dad is 39 years old.
Solve each equation for $x$. Check your solution.

1. $x - 4 = 27$
2. $-4x = 84$
3. $6x = 54$
4. $-12 + x = -30$
5. $96 = 35 + x$
6. $\frac{x}{5} = 8$
7. $\frac{x}{6} = -12$
8. $x - 3 = -9$
9. $-110 = -5x$

10. Shawn did not check his solutions for the four-problem quiz on one-step equations. Check Shawn’s answers. If the answer is incorrect, find the correct answer.

   a. $x + 82 = 124$
      Shawn’s answer: $x = 206$

   b. $-7 + x = 12$
      Shawn’s answer: $x = 19$

   c. $9x = 63$
      Shawn’s answer: $x = -7$

   d. $\frac{x}{4} = 8$
      Shawn’s answer: $x = 2$

11. Patty wants to use an equation mat, integer chips and variable cubes to illustrate a one-step multiplication equation. On a separate piece of paper, draw a picture of how she could illustrate the equation $3x = 12$ on the mat. Illustrate how the integer chips can be separated into three equal parts on the mat to show the solution to the equation. State the value of $x$.

Write an equation for each statement and solve for the variable. Check your solution.

12. The product of 8 and a number is 48.

13. The sum of a number and $-53$ is 89.

14. A number divided by $-7$ is $-7$.

15. Sixteen less than a number is 102.

16. A number increased by $1\frac{1}{2}$ is $2\frac{3}{4}$.

17. One-third of a number is 6.
18. Mike is thinking of two numbers. Their difference is 12. If one of the numbers is 19, what is the other number? Is there only one answer? If not, what is the other possibility for the second number?

19. It is a common belief that one human year is equal to 7 dog years.
   a. Write an expression that will calculate how old a dog is in dog years based on its normal “human year” age. Use $x$ to represent the number of human years.
   b. If a dog is 84 dog years old when he passes away, how old was he in human years? Write an equation to represent this situation and solve the equation.

Solve each equation for $x$. Check the solution.

20. $x + 1.73 = 2.81$  
21. $-0.1x = 16$  
22. $\frac{2}{3}x = 10$

23. $\frac{6}{11}x = \frac{21}{22}$  
24. $-2.5 + x = 6.75$  
25. $-x = 3.4$

**REVIEW**

Evaluate each expression.

26. $12 + 12 ÷ 3 \cdot 5 – 1$  
27. $\frac{(3 + 7)^2}{5} + 6$

28. $3(-21 + 10) + 7$  
29. $-4\left(\frac{1}{2} + \frac{1}{2}\right) - 2$

Use the Distributive Property to simplify each expression.

30. $6(x - 3)$  
31. $-2(9x - 1)$

32. $-4(5m + 7)$  
33. $\frac{1}{4}(8x - 4)$

34. Evaluate the following using mental math.
   a. $-3 + -2$  
   b. $-4(6)$  
   c. $-5 + 11$  
   d. $6 - 9$

   e. $27 ÷ 3$  
   f. $(-20)(-5)$  
   g. $-8 + 1$  
   h. $\frac{-36}{6}$
Jim took a summer road trip across the country. He started his trip 20 miles east of Portland. He headed east on I-84. Every hour \((h)\) he traveled 60 miles further away from Portland. The expression that represents his distance from Portland is:

\[
d = 20 + 60h
\]

Jim forgot to bring a watch, but noticed that he was 500 miles from Portland after his first day of driving. Assuming Jim traveled at an equal rate, how many hours did Jim drive on the first day of his road trip?

To solve this problem, you must be able to solve a two-step equation. Since Jim has driven 500 miles, substitute that value for \(d\) in the equation:

\[
500 = 20 + 60h
\]

To isolate the variable in a two-step equation you must perform two inverse operations. The inverse operations must undo the order of operations. That means you start by undoing addition or subtraction. Then use inverse operations to remove any multiplication or division.

**How long has Jim been on the road?**

Write an equation. Subtract 20 from both sides of the equation. Divide both sides of the equation by 60.

\[
\begin{align*}
500 &= 20 + 60h \\
-20 &= -20 \\
480 &= 60h \\
60 &= 60h \\
8 &= h
\end{align*}
\]

Jim has been on his road trip for 8 hours.

**EXAMPLE 1** Solve the equation for \(x\). Check the solution. \(8x - 3 = 85\)

**Solution**

Add 3 to both sides of the equation. Divide both sides of the equation by 8.

\[
\begin{align*}
8x - 3 &= 85 \\
+3 &= +3 \\
8x &= 88 \\
\frac{8x}{8} &= \frac{88}{8} \\
x &= 11
\end{align*}
\]

\(✓\) Check the answer.

\[
\begin{align*}
8(11) - 3 &= 85 \\
88 - 3 &= 85 \\
85 &= 85
\end{align*}
\]
Lesson 5 ~ Solving Two-Step Equations

**Example 2**

**Solution**

Solve the equation for \( x \). Check the solution.

\[-2 = \frac{x}{9} + 5\]

Subtract 5 from both sides of the equation.

\[-2 + \frac{x}{9} + 5\]

Multiply both sides of the equation by 9.

\[9 \cdot (-7) = x \cdot 9\]

\[-63 = x\]

☑ Check the solution.

\[-2 \neq \frac{-63}{9} + 5\]

\[-2 \neq -7 + 5\]

\[-2 = -2\]

**Example 3**

Use an equation mat to illustrate and solve the equation \( 2x - 4 = 6 \).

**Solution**

Lay out the variable cubes and integer chips to match the equation.

Remove the integer chips from the side with the variable by canceling out four negative integer chips with four positive integer chips (add 4 positive chips to each side).

Divide the integer chips on the right side of the mat equally between the two cubes.

Write the solution.

\( x = 5 \)
EXAMPLE 4

Eight more than 3 times a number is 29. Write and solve an equation to find the value of the number.

SOLUTION

Write the equation.
Three times a number
Eight more than
Is 29

\[ 3x + 8 = 29 \]

\[ \frac{3x + 8}{3} = \frac{29}{3} \]

\[ x = 7 \]

☑ Check the solution.

\[ 3(7) + 8 = 29 \]
\[ 21 + 8 = 29 \]
\[ 29 = 29 \]

EXERCISES

Solve each equation for \( x \). Check the solution.

1. \( 4x + 2 = 18 \)

2. \( \frac{x}{3} - 4 = 26 \)

3. \( \frac{x}{5} - 7 = 3 \)

4. \( 102 = 20x - 8 \)

5. \( -8 + 10x = 102 \)

6. \( \frac{1}{2}x + \frac{3}{4} = 1 \)

7. \( -18 = \frac{x}{3} + 2 \)

8. \( 2.5x + 7.5 = 20 \)

9. \( 32 = -8 + 4x \)

10. The Fahrenheit and Celsius scales are related by the equation: \( F = \frac{9}{5}C + 32 \)

a. The lowest temperature in Alaska, \(-62^\circ\) C, was recorded on January 23, 1971 at Prospect Creek Camp. Use the formula to convert the record temperature to Fahrenheit.

b. In February, the average high temperature in Puerto Vallarta, Mexico is \(81^\circ\) F. Use the formula to convert the average temperature to Celsius.
11. Jordin solved three problems incorrectly. Describe the error she made in each problem; then find the correct answers.

a. \[3x - 6 = 27\]
   \[-6 \quad -6\]
   \[\frac{3x}{3} = \frac{21}{3}\]
   \[x = 7\]

b. \[10x + 15 = 45\]
   \[-15 \quad -15\]
   \[\frac{10x}{5} = \frac{30}{5}\]
   \[2x = 6\]
   \[x = 3\]

c. \[\frac{x}{8} = 22 + 2\]
   \[\frac{8}{8} = \frac{24}{8}\]
   \[x = 3\]

Write an equation for each statement. Solve each equation and check the solution.

12. Seven more than twice a number is 25.

13. Six less than the quotient of a number and 3 is −1.

14. Twelve decreased by 5 times a number is 72.

15. Four more than one-half a number is 3.

16. Barry begins the year with $25 in his piggy bank. At the end of each month, Barry adds $3.
   a. How much will Barry have after 5 months have passed?
   b. Write a formula that could be used to calculate Barry’s total savings (S) based on how many months (m) he has deposited money in his bank.
   c. Use your formula to determine how many months have passed when Barry reaches $100 in his account.

17. Mariah’s parents got into a car accident. They took their car to the shop to be repaired. When the car was finished, they received a bill of $637. The total cost for parts was $280 and the cost of labor was $42 per hour. Determine how many hours the mechanics spent working on the car. Explain in words how you found your answer.

18. Ryan sells cars for a living. He gets paid $50 per day plus a commission of 2% of the total cost of each car he sells. Ryan’s daily earnings (E) can be represented by the equation \[E = 0.02x + 50\] where \(x\) is the amount of his sales for the day.
   a. What does the 0.02 represent in the equation?
   b. How much did Ryan make on Monday if he only sold one car for $4,400?
   c. Ryan wants to make $1,000 in a day. What must his daily sales be to reach this goal?
Lesson 5 ~ Solving Two-Step Equations

Copy each line and insert any combination of the four operations (+, −, ×, ÷) to make each statement true.

19. \(2 \quad 3 \quad 5 = 17\)  
20. \(12 \quad 6 \quad 10 = -8\)

21. \(-1 \quad 2 \quad 2 = 0\)  
22. \(20 \quad 5 \quad 50 \quad 5 = 45\)

Simplify each expression.

23. \(9(x - 2)\)  
24. \(-7 + 6(x - 5) + 2x\)

25. \(2x + 4x + 7x - 3x\)  
26. \(2(x - 1) + 3(x + 1)\)

27. \(4(x + 8) - 12\)  
28. \(-5(x + 3) + 5x + 13\)

**Tic-Tac-Toe ~ Temperature Systems**

There are three common units of temperatures: Fahrenheit, Kelvin and Celsius.

1. Research the three common units of temperature. Where was each one invented? Where is each one most often used? How are the units related to each other?

The following relationships are used to convert one unit of temperature to another:

\[F = 1.8C + 32\]  
\[K = C + 273.15\]

2. Convert the following units from one system to another.
   a. 20° C to Fahrenheit  
   b. 20° C to Kelvin
   c. 78° F to Celsius  
   d. 289.5 K to Celsius
   e. -4° F to Celsius  
   f. 315 K to Fahrenheit

3. Develop an equation that will convert Fahrenheit to Kelvin.

**Tic-Tac-Toe ~ What’s the Process?**

Solving a multi-step equation can be a complicated task depending on the difficulty of the equation. Create a poster to help classmates through the process step-by-step. Include at least two examples on your poster. Address what a student should do if the following items show up in their equation:

- Multiplication or Division
- Addition or Subtraction
- Variables on Different Sides of the Equals Sign
- Variables on the Same Side of the Equals Sign
- Parentheses
Two different stores at the beach rent bicycles. One store charges an initial fee of $4 plus $2 per hour. This can be represented by the expression $4 + 2h$ when $h$ is the number of hours the bike is rented. The other store charges an initial fee of $10 but only charges $0.50 per hour. This situation is represented by the expression $10 + 0.5h$ when $h$ is the number of hours the bike is rented. At what point would the two bike rentals cost the exact same amount?

The times would be the same when the two expressions are equal.

$$4 + 2h = 10 + 0.5h$$

To solve this equation, you must first get the variables on the same side of the equation. To do this, move one variable term to the opposite side of the equation using inverse operations. It is easiest to move the variable term with the smaller coefficient. This often helps you deal with fewer negative numbers. After the variables are on the same side of the equation, solve the two-step equation that remains.

Subtract $0.5h$ from both sides of the equation.

$$4 + 2h - 0.5h = 10 + 0.5h - 0.5h$$

$$4 + 1.5h = 10$$

Subtract 4 from both sides of the equation.

$$4 + 1.5h - 4 = 10 - 4$$

$$1.5h = 6$$

Divide both sides of the equation by 1.5.

$$\frac{1.5h}{1.5} = \frac{6}{1.5}$$

$$h = 4$$

The two bike rentals cost the same amount at 4 hours.

To check the solution, substitute the value into each side of the equation. If both sides are equal, the solution is correct.

First store for 4 hours: $\checkmark$ $4 + 2(4) = 12$

Second store for 4 hours: $\checkmark$ $10 + 0.5(4) = 12$

**Solving Multi-Step Equations**

1. Simplify each side of the equation by distributing and combining like terms, when necessary.
2. If variables are on both sides of the equation, balance the equation by moving one variable term to the opposite side of the equals sign using inverse operations.
3. Follow the process for solving one- and two-step equations to get the variable by itself.
Step 1: Write the equation that is represented by the equation mat shown below.

Step 2: There are variable cubes on both sides of the mat. Remove the same number of cubes from each side so that only one side has variable cubes remaining. Draw a picture of what is on the mat now.

Step 3: What is the next step you must take to balance the equation mat? Remember that your goal is to isolate the variable. Draw a picture of what is on the mat now.

Step 4: Once the variables are isolated, $x$ can be determined by dividing the integer chips on the opposite side equally between the remaining variable cubes. How many integer chips belong to each variable cube? What does this represent?

Step 5: Draw a representation of $5x + 7 = 2x + 4$ on an equation mat.

Step 6: Repeat Steps 2 - 4. What is the solution to this equation?

**EXAMPLE 1** Solve the equation for $x$. $4(2x - 7) = 20$

**Solution**

Distribute. $4(2x - 7) = 20$

Add 28 to both sides of the equation. $8x - 28 = 20 + 28$

Divide both sides of the equation by 8. $8x = 48$

$x = 6$

✓ Check the solution. $4(2 \cdot 6 - 7) \div 20$

$4(12 - 7) \div 20$

$4(5) \div 20$

$20 = 20$
Lesson 6 ~ Solving Multi-Step Equations

**EXAMPLE 2**

**Solve the equation for** $x$.  
$-2x + 9 = 4x - 15$

**Solution**

There are no parentheses so there is no need to use the Distributive Property.

Add $2x$ to both sides of the equation.

\[
-2x + 9 + 2x = 4x - 15 + 2x
\]

Add 15 to both sides of the equation.

\[
9 + 15 = 6x - 15 + 15
\]

Divide both sides of the equation by 6.

\[
\frac{24}{6} = \frac{6x}{6}
\]

\[
4 = x
\]

☐ Check the solution.

\[
-2(4) + 9 = 4(4) - 15
\]

\[
-8 + 9 = 16 - 15
\]

\[
1 = 1
\]

**EXAMPLE 3**

**Solve the equation for** $x$.  
$5(x + 8) = -2(x - 13)$

**Solution**

Use the Distributive Property to remove the parentheses.

\[
5(x + 8) = -2(x - 13)
\]

\[
5x + 40 = -2x + 26
\]

Add $2x$ to both sides of the equation.

\[
+2x + 2x = 26
\]

Subtract 40 from both sides of the equation.

\[
\frac{-40}{7} = \frac{-7x}{7}
\]

\[
x = -2
\]

☐ Check the solution.

\[
5(-2 + 8) = -2(-2 - 13)
\]

\[
5(6) = -2(-15)
\]

\[
30 = 30
\]

**EXAMPLE 4**

Katie opened a coffee cart to earn some extra money. Her one-time equipment start-up cost was $460. It costs her $1 to make each cup of coffee. She plans to sell the cups of coffee for $3. How many cups will she need to sell before she breaks even?

**Solution**

Let $x$ represent the number of cups of coffee sold.

Write an equation that represents the situation.

\[
460 + 1x = 3x
\]

Start Up  Cost per cup  Amount charged per cup
EXERCISES

1. Draw an equation mat with integer chips and variable cubes that represents the equation $x + 9 = 3x + 1$. Describe in words or pictures how you would move items around on the mat to solve this equation. What is the solution to this equation?

Solve each equation. Check the solution.

2. $2(3x + 6) = 42$
3. $10x = 4x + 66$
4. $-3(x + 4) = -15$
5. $9x + 20 = 34 - 5x$
6. $5x + 3 - 2x = 3$
7. $\frac{1}{2}(6x - 8) = 41$

8. Explain the steps to finding the solution for a multi-step equation.

9. Kelsey made cupcakes to sell as a fundraiser for her trip to Washington DC. She purchased one cupcake pan for $12. She calculated that the ingredients cost her $0.25 per cup cake. She plans to sell the cupcakes for $0.75 each.
   a. Write an equation that could be used to find the number of cupcakes ($x$) she will need to sell to break even.
   b. Solve the equation from part a. Check the solution.
   c. If she sold 184 cupcakes, what was her profit?

Solve each equation. Check the solution.

10. $7x - 9 = -4x + 90$
11. $\frac{1}{4}x + 12 = \frac{1}{2}x + 10$
12. $3(x - 6) = 6x - 90$
13. $2x + 7x + 10 = 11x$
14. $6(x + 2) = 2(2x + 4)$
15. $4.6x - 8.5 = 1.3x + 1.4$
16. $5x = 10x - 4x + 7$
17. $3(2x - 5) = 2x + 1$
18. $x - \frac{1}{3} = 3x + \frac{2}{3}$

Katie must sell 230 cups of coffee before she will break even.
19. Francisco made only one mistake on his homework. Describe the mistake he made and solve the equation correctly.

\[
\begin{align*}
3(x - 7) &= 5x - 11 \\
3x - 7 &= 5x - 11 \\
-3x &= -3x \\
-7 &= 2x - 11 \\
+11 &= +11 \\
4 &= 2x \\
2 &= x
\end{align*}
\]

20. An internet movie rental company has two different options for renting movies.
   a. Copy the table below and fill in the total amount paid for movies rented under each plan.

<table>
<thead>
<tr>
<th>Movies Rented</th>
<th>Total Cost Option A</th>
<th>Total Cost Option B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>$40</td>
<td>$0</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

   b. Write an expression to represent the total cost for **Option A**. Use x for the number of videos rented.
   c. Write an expression to represent the total cost for **Option B**. Use x for the number of videos rented.
   d. Set the two expressions equal to each other. Solve for x.
   e. Explain what the answer to part d means in real life. Does the table in part a support your answer?
   f. Sal thinks he will rent about 15 movies this year. Which plan would be a better deal for him? Why?

**REVIEW**

State whether each equation is true or false for the values of the variables given.

21. \(5x - 2y = 10\) when \(x = 2\) and \(y = 0\)
22. \(4x + 1 = y\) when \(x = 5\) and \(y = 46\)
23. \(2(x + 3) = y\) when \(x = 3\) and \(y = 9\)
24. \(y = -4(x + 10)\) when \(x = -6\) and \(y = -16\)
When someone opens a small business, they often use linear equations to determine profits and losses. Three individuals started up three different businesses which are described below.

### Business #1
Sarah's Coffee Shop
- Start-Up Cost = $275
- Profit per Coffee Sold = $2.50
- Linear Equation:
  \[ P = -275 + 2.50C \]

### Business #2
Jason's Skateboarding Store
- Start-Up Cost = $400
- Profit per Board Sold = $14

### Business #3
Jamal's Gaming Shop
- Start-Up Cost = $750
- Profit per Game Sold = $9.75

1. A linear equation representing total profits is shown for Business #1. Write the linear equations for the total profits for the other two businesses.
2. For each business, determine how many items they will need to sell to earn back the amount of money they spent for starting their business.
3. Each business has a goal of making a total profit of $10,000 in the first quarter of the year. Determine how many total items they will each need to sell to reach their goal.
4. Design your own small business.
   a. Choose one item to sell. Describe why you would like to sell this item.
   b. Estimate the total start-up costs. Explain how you came up with this amount.
   c. Estimate the total profit you hope to make per item sold. Explain how you came up with this amount.
   d. Repeat #1 - #3 above for your small business idea.
   e. Do you think there would be enough interest in your product to reach the goal of $10,000 in the first quarter? Explain.

### Tic-Tac-Toe ~ Equation Mats

Equation mats are used to see a visual model of the equation-solving process. Write a “How To…” guide about using equation mats for different types of equations. Include the following types of equations in your guide:
- One-Step Equations
- Two-Step Equations
- Equations with Variables on Both Sides of the Equals Sign
The coordinate plane is created by drawing two number lines which intersect at a 90° angle. These two lines are called the axes. Each number line intersects the other at zero. The point where the two lines cross is called the origin. The horizontal axis is used for the variable \(x\) (called the \(x\)-axis) and the vertical axis is used for the variable \(y\) (called the \(y\)-axis). The axes divide the coordinate plane into four quadrants. The quadrants are numbered I, II, III and IV starting in the top right quadrant and moving counter-clockwise.

Each point on the graph is named by an ordered pair. The first number in the ordered pair corresponds to the numbers on the \(x\)-axis. The second number corresponds to the number on the \(y\)-axis.

**EXAMPLE 1**

Graph each point and name the quadrant where each point is located.

a. \(A(5, 6)\)  
b. \(B(-4, 8)\)  
c. \(C(0, 3)\)  
d. \(D(9, -1)\)

**SOLUTIONS**

**Quadrant Location**

a. Quadrant I  
b. Quadrant II  
c. None (on the \(y\)-axis)  
d. Quadrant IV

\[(-4, 8) \rightarrow \text{Start at the origin and move left 4 units and up 8 units.}\]

\[(5, 6) \rightarrow \text{Start at the origin and move right 5 units and up 6 units.}\]

\[(9, -1) \rightarrow \text{Start at the origin and move right 9 units and down 1 unit.}\]
Xavier was babysitting his little sister, Sam, one afternoon. His sister wanted to do a connect-the-dots but Xavier could not find any in the house. He decided to create one for Sam to do. He recorded the ordered pairs that would need to be connected in the order he wanted Sam to connect them.

**Step 1:** Draw and label the x- and y-axis. Number each axis from −10 to 10.

**Step 2:** Plot Xavier’s points and connect the points in the order they are listed.

Start at (8, 0) → (6, 2) → (−2, 2) → (−5, 5) → (−5, −5) → (−2, −2) → (6, −2) → (8, 0)

**Step 3:** Is the picture recognizable? If so, what do you think it is?

**Step 4:** What are a few of the limitations of trying to draw a picture by connecting the dots on a coordinate plane?

**Step 5:** Create your own instructions for a connect-the-dots that uses at least eight points in at least three quadrants. List your points in the order they should be connected.

**Step 6:** Have a classmate try your connect-the-dots. Does it look like what you expected?

Real-life data can be examined on a coordinate plane to look for relationships between the data. A scatter plot is one way to determine if two quantities are related. A scatter plot is a set of ordered pairs graphed on a coordinate plane where each ordered pair represents two data values. Once data can be seen visually on a scatter plot, it is possible to see whether or not there is a relationship between the two sets of data. Examples of data sets that might be examined on a scatter plot are height and weight, years of schooling and salary, or average temperature and latitude placement.

**EXAMPLE 2**

Derrick surveyed eight friends to determine if there is a relationship between height and shoe size.

a. Make a scatter plot of the data. Put height on the x-axis and shoe size on the y-axis.

<table>
<thead>
<tr>
<th>Height (inches), x</th>
<th>60</th>
<th>72</th>
<th>65</th>
<th>63</th>
<th>74</th>
<th>69</th>
<th>64</th>
<th>59</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shoe Size, y</td>
<td>7</td>
<td>12</td>
<td>9</td>
<td>8</td>
<td>14</td>
<td>9</td>
<td>7</td>
<td>5</td>
</tr>
</tbody>
</table>

b. In general, how does shoe size change as height changes?

a. Plot each pair of data values.

b. The scatter plot shows that as height increases, shoe size appears to increase.
EXAMPLE 3  
Draw a scatter plot to model the data given below. Describe the pattern seen in the scatter plot.

<table>
<thead>
<tr>
<th>Year</th>
<th>Cost of First Class Stamp (in cents)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985</td>
<td>22</td>
</tr>
<tr>
<td>1988</td>
<td>25</td>
</tr>
<tr>
<td>1991</td>
<td>29</td>
</tr>
<tr>
<td>1995</td>
<td>32</td>
</tr>
<tr>
<td>1999</td>
<td>33</td>
</tr>
<tr>
<td>2001</td>
<td>34</td>
</tr>
<tr>
<td>2002</td>
<td>37</td>
</tr>
<tr>
<td>2006</td>
<td>39</td>
</tr>
<tr>
<td>2007</td>
<td>41</td>
</tr>
<tr>
<td>2008</td>
<td>42</td>
</tr>
</tbody>
</table>

As time passes, stamp prices appear to rise steadily.

SOLUTION

EXERCISES

1. Draw a coordinate plane with an $x$-axis and $y$-axis that go from $-10$ to $10$. Label each axis, quadrant and the origin.

2. On the coordinate plane drawn in Exercise 1, graph and label the following ordered pairs.
   - $A(4, 7)$  
   - $B(-4, -1)$  
   - $C(2, 0)$  
   - $D(0, -9)$  
   - $E(-1, 5)$

3. Give the ordered pair for each point on the coordinate plane shown.
Tell whether each point described below is on the \(x\)-axis, \(y\)-axis or in Quadrant I, II, III or IV.

4. The \(x\)-coordinate is positive and the \(y\)-coordinate is negative.

5. The \(x\)-coordinate is 0 and the \(y\)-coordinate is positive.

6. Both coordinates are positive.

7. The \(x\)-coordinate is negative and the \(y\)-coordinate is 0.

8. Both coordinates are negative.

9. Mr. Harrison wanted to see if there was a relationship between the number of missing homework assignments his students had from the last chapter and how the students did on the chapter test. The data he collected is shown in the table below.

<table>
<thead>
<tr>
<th>Number of Missing Assignments</th>
<th>2</th>
<th>0</th>
<th>4</th>
<th>6</th>
<th>1</th>
<th>3</th>
<th>4</th>
<th>0</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grade on Chapter Test</td>
<td>82%</td>
<td>96%</td>
<td>74%</td>
<td>50%</td>
<td>94%</td>
<td>80%</td>
<td>68%</td>
<td>100%</td>
<td>90%</td>
</tr>
</tbody>
</table>

a. Draw a scatter plot to model the data. Put the number of missing assignments on the \(x\)-axis and the students' test scores on the \(y\)-axis.

b. Describe the pattern seen in the scatter plot. Do you think this relationship is true for most students in your classes? Why or why not?

10. Brittany told her friend, “I think that as people get older, they don’t watch as many movies.” In order to verify her statement, she asked ten different people how many movies they had seen during the past month. Her survey results are shown on the scatter plot.

a. Based on the data collected, would you agree or disagree with Brittany’s statement? Defend your answer.

b. Based on the scatter plot, could you predict how many movies a fifty-year-old watches? If so, how many? If not, why?
11. Jillian went for a run on the treadmill. Every so often she recorded her heart rate.

<table>
<thead>
<tr>
<th>Minutes run</th>
<th>Heart Rate (beats per minute)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>96</td>
</tr>
<tr>
<td>8</td>
<td>118</td>
</tr>
<tr>
<td>13</td>
<td>130</td>
</tr>
<tr>
<td>20</td>
<td>144</td>
</tr>
<tr>
<td>25</td>
<td>156</td>
</tr>
<tr>
<td>28</td>
<td>160</td>
</tr>
<tr>
<td>34</td>
<td>162</td>
</tr>
<tr>
<td>40</td>
<td>164</td>
</tr>
</tbody>
</table>

a. Draw a scatter plot to model the data she gathered.
b. Describe the pattern you see in the scatter plot.
c. What would you predict her heart rate would be after an hour of running? Why?

**REVIEW**

Solve each equation for $x$. Check the solution.

12. $5x - 7 = 23$
13. $17 = \frac{1}{2}(2x + 24)$
14. $-6(x - 1) = 66$
15. $4x + 3 = 3x + 14$
16. $7 = -2 + 2x$
17. $\frac{x}{3} - 1.6 = 5.8$
18. $-x + 9 = 2$
19. $3x + 1 = 11 - 2x$

**Tic-Tac-Toe ~ Scatter Plot Survey**

Scatter plots are used to show correlation between two items.

1. Choose one of the following pair of survey questions listed below. Collect responses from at least 15 individuals. Make sure you ask people who represent a variety of different groups (age, height, careers, etc). Record your data in a table.
   A. How tall are you (in inches)? What is your shoe size?
   B. What is your favorite number? How many years of school have you attended?
   C. How old are you? How many television shows do you watch each week?

2. Determine which item will be represented by the variable $x$ and which item will be represented by $y$. Explain how you made your decision.

3. Display your data in a scatter plot drawn on graph paper.

4. Based on your survey, do you believe the two items are correlated? Does this surprise you? Explain your answer.
A mathematical sentence that contains >, <, ≥ or ≤ is an inequality. A solution to an inequality is any value that makes the inequality true.

**For example:** Given that \( x \geq -1 \), one solution to the inequality is 4 because 4 ≥ −1.

Solutions to an inequality can be graphed on a number line. When using the > or < inequality symbols, an open circle is used on the number line because the solution does not include the given number. For example, if \( x > 2 \), the solution cannot include 2 because 2 is not greater than 2. When using the ≥ or ≤ inequality symbols, a closed (or filled in) circle is used because the solution contains the given number.

Determining which direction the arrow should point is based on the relationship between the variable and the solution. For example, if \( x \geq -1 \) then the arrow will point to all numbers greater than −1.

Inequalities are solved using properties similar to those you used to solve equations. Use inverse operations to isolate the variable so the solution can be graphed on a number line.

One special rule applies to solving inequalities. Whenever you multiply or divide by a negative number on both sides of the equation, you must flip the inequality symbol. For example, < would become > if you multiply or divide by a negative when performing inverse operations.

**For example:**

\[
-4x + 7 \leq 19
\]

\[
\begin{align*}
-4x + 7 & \leq 19 \\
-7 & \quad -7 \\
\frac{-4x}{-4} & \quad \frac{12}{-4} \\
x & \geq -3
\end{align*}
\]

Reverse the inequality symbol when dividing by a negative.

Solve each inequality below. Graph the solution on a number line.

1. \( 2x + 7 > 15 \)
2. \( \frac{x}{2} - 1 \geq -4 \)
3. \( -3x - 4 < 5 \)
4. \( 5(x + 3) \leq 20 \)
5. \( 7 > \frac{x}{4} + 6 \)
6. \( 9x < 2x - 35 \)
7. \( -7 + 4x \geq 3 - 6x \)
8. \( 2(x + 3) \geq 5x + 12 \)
9. \( \frac{1}{2}x + 8 < x + 4 \)
Lesson 1 ~ Order of Operations

Evaluate each expression.

1. \(6 + 7 \cdot 3 - 9\)
2. \(35 \div 7 \cdot 2 - 4^2\)
3. \(|4 - 8| + |8 - 1|\)
4. \((-2 + 5)^2 - 10\)
5. \(2(11 - 5) - 10\)
6. \(\frac{9 + 11}{1 + 1^2}\)
7. \(\frac{-14 - 4}{2 + 1} - 5\)
8. \(\frac{7|11 - 4| + 1}{(-5)^2}\)
9. \(\frac{1}{2}(6 + 8) - 2(3 - 5)\)

Lesson 2 ~ Evaluating Expressions

Evaluate each expression for the given values of the variables.

10. \(2x + 3\) when \(x = 4\)
11. \(12 - 3f\) when \(f = 1.2\)
12. \(\frac{2y - 1}{2y}\) when \(y = 3\)
13. \(10(p + 5)\) when \(p = -7\)
14. \(6a^2 - 14\) when \(a = 3\)
15. \(7m + 1\) when \(m = \frac{1}{2}\)
Copy each table. Complete each table by evaluating the given expression for the values listed.

16. \[
\begin{array}{|c|c|c|}
\hline
x & 5x + 1 & \text{Output} \\
\hline
-2 & & \\
0 & & \\
\frac{1}{2} & & \\
3 & & \\
7 & & \\
\hline
\end{array}
\]

17. \[
\begin{array}{|c|c|c|}
\hline
x & \frac{3x + 2}{4} & \text{Output} \\
\hline
-3 & & \\
0 & & \\
4 & & \\
10 & & \\
\hline
\end{array}
\]

Lesson 3 ~ The Distributive Property

Use the Distributive Property to simplify each expression.

18. \[5(x + 4)\]
19. \[\frac{1}{4}(8x - 2)\]
20. \[-1(3x - 12)\]
21. \[6(3x - 10)\]
22. \[-3(-8x + 5)\]
23. \[0.1(15x + 5)\]

Simplify each expression.

24. \[3x + 7 + 4x - 2\]
25. \[3(x - 4) + 6\]
26. \[5(x - 1) - 5\]
27. \[3(x - 4) + 2(x + 1)\]
28. \[4x + 3x - 5x + 2 - x\]
29. \[6x + 40 + 4x - 15\]

Write and simplify an expression for the perimeter of each figure.

30. \[
\begin{array}{c}
8x \\
6x \\
\end{array}
\]

31. \[
\begin{array}{c}
x - 4 \\
x - 4 \\
x - 4 \\
x - 4 \\
x - 4 \\
\end{array}
\]

Lesson 4 ~ Solving One-Step Equations

Solve each equation. Check the solution.

32. \[x + 13 = 35\]
33. \[\frac{x}{7} = -3\]
34. \[10x = 90\]
35. \[-8x = -44\]
36. \[\frac{1}{2}x = 10\]
37. \[3.7 = \frac{x}{6}\]

Write an equation for each statement. Solve each equation. Check the solution.

38. The sum of seven and a number is 73.
39. Ten less than a number is 66.
40. Twenty-nine is equal to a number divided by three.
Lesson 5 ~ Solving Two-Step Equations

Solve each equation. Check the solution.

41. \(10x - 8 = 92\)  
42. \(-5x - 1 = 44\)  
43. \(4 = \frac{x}{6} - 2\)

44. \(\frac{x}{7} + 1.3 = 2.1\)  
45. \(2x - 7 = 4\)  
46. \(\frac{1}{3}x + 3 = 3\)

47. Kari is saving money for an MP3 player. She began the year with $30 of savings. At the end of each month, Kari adds $14.
   a. How much will Kari have after 3 months have passed?
   b. Write a formula to calculate Kari's total savings (S) based on how many months (m) she has saved this year.
   c. The MP3 player Kari wants costs $170. Use your formula to determine how many months it will take for Kari to have enough to purchase the MP3 player.

Lesson 6 ~ Solving Multi-Step Equations

Solve each equation. Check the solution.

48. \(6(x + 3) = 42\)  
49. \(5x + 2 = 3x - 2\)  
50. \(-4x + 9 = x - 11\)

51. \(2(x - 1) = 3x - 13\)  
52. \(\frac{1}{4}(8x + 1) = 12 \frac{1}{4}\)  
53. \(9x + 4x - 7 = 3x - 17\)

54. A bowling alley has two payment options for bowlers. Option A allows bowlers to pay $3 per game. Option B allows bowlers to join the 'Elite Bowling Club' for $15 plus an additional $1.50 per game.
   a. Write an expression for the cost of \(x\) games with Option A.
   b. Write an expression for the cost of \(x\) games with Option B.
   c. Set the two expressions equal to one another. Solve the equation. How many games would someone need to bowl to make both options cost the same amount?
   d. If Ray is going to bowl 14 games, which option should he choose? Why?

55. Julie had $400 in her savings account at the beginning of the summer. Each week she took $20 out of her account. Stephen had $100 in his account at the beginning of summer. Each week he added $30 to his account.
   a. Write an expression for the amount in Julie's account after \(x\) months.
   b. Write an expression for the amount in Stephen's account after \(x\) months.
   c. Set the two expressions equal to each other. Solve the equation to determine how many weeks it took for Julie and Stephen to have the same amount in their accounts.
Lesson 7 ~ The Coordinate Plane and Scatter Plots

For problems 56 - 59 give the ordered pair for each point on the coordinate plane shown below.

56. A
57. B
58. C
59. D

60. Give the coordinates of a point that could be found in Quadrant II.
61. Give the coordinates of a point that could be found on the y-axis.
62. Give the coordinates of a point that could be found in Quadrant IV.
63. Give the coordinates of the origin.

64. Jason noticed that there were lots of people at the park on a warm day. He decided to collect data comparing the outside temperature to the number of people at his neighborhood park on different days.
   a. Make a scatter plot of the data. Put temperature on the x-axis and the number of people at the park on the y-axis.

<table>
<thead>
<tr>
<th>Temperature (F°)</th>
<th>60°</th>
<th>72°</th>
<th>86°</th>
<th>52°</th>
<th>74°</th>
<th>80°</th>
<th>66°</th>
<th>78°</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of People at Park</td>
<td>6</td>
<td>8</td>
<td>14</td>
<td>3</td>
<td>9</td>
<td>10</td>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>

b. Is there a relationship between the weather outside and the number of people at the park? If so, what is the relationship?
c. Based on the scatter plot, predict the number of people at the park when it is 100° outside. Do you think this is realistic? Why or why not?
Rodger  
**Insurance Agency Owner**  
**Baker City, Oregon**

I am the owner of an insurance agency that deals mostly with farmers. We consult with clients to help them understand how to protect themselves from disaster by having the right kind of insurance. We also offer prices for whatever coverage a client may need. If a customer decides to buy insurance, we deliver their policy and explain it to them. Our company tries to give excellent service so our clients will want to stay with us for a long time.

The amount of money a client must pay for insurance is determined by math formulas. Most of the time, calculations are done by computers. Some insurances, though, are still done without computers. One type of insurance where we do the math ourselves is crop insurance. If a farmer tells us he wants to get insurance on 500 tons of hay, we have to determine a couple of things. The first question to answer is how much they want the hay insured for. The next question is for how long they would like the hay covered. There is a table that gives all of the different rates of policies depending on those two questions. We can use the rates from the table to come up with a price for the farmer’s policy.

Insurance agents also use math to determine fire insurance rates. An insurance agent has to add all the different costs that would go into rebuilding a house if it were to burn down. Some of these calculations can become pretty complicated. In many ways, it is just like a big story problem.

You do not need a college education to work in the insurance field, but it is a good idea to have one. There are many different career paths in insurance, and a college education will help you in whatever path you choose to follow.

Insurance is commissions-based. This means that you make a percentage of whatever amount of insurance you sell. The more you sell, the more money you make. The harder you work, the more you are rewarded.

I like being an insurance agency owner. As the owner I do a little bit of everything. I fix computers, answer phones, meet with clients and pay the employees and bills. No day is the same, and no day is ever a slow day. I also like the fact that the effort you put into the work affects how much you earn. Not all jobs are like that. Lastly, I like my career because it gives me a very satisfying feeling to know that I helped someone out when they encountered a situation in life that required insurance.